RECOGNISING ACHIEVEMENT

ADVANCED GCE **4731/01**

MATHEMATICS

Mechanics 4

WEDNESDAY 18 JUNE 2008 Morning

Time: 1 hour 30 minutes

Additional materials (enclosed): None

Additional materials (required):
Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is **72**.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

1 Two flywheels F and G are rotating freely, about the same axis and in the same direction, with angular speeds $21\,\text{rad s}^{-1}$ and $36\,\text{rad s}^{-1}$ respectively. The flywheels come into contact briefly, and immediately afterwards the angular speeds of F and G are $28\,\text{rad s}^{-1}$ and $34\,\text{rad s}^{-1}$, respectively, in the same direction. Given that the moment of inertia of F about the axis is $1.5\,\text{kg m}^2$, find the moment of inertia of G about the axis. [4]

2 A rotating turntable is slowing down with constant angular deceleration. It makes 16 revolutions as its angular speed decreases from $8\,\text{rad s}^{-1}$ to rest.

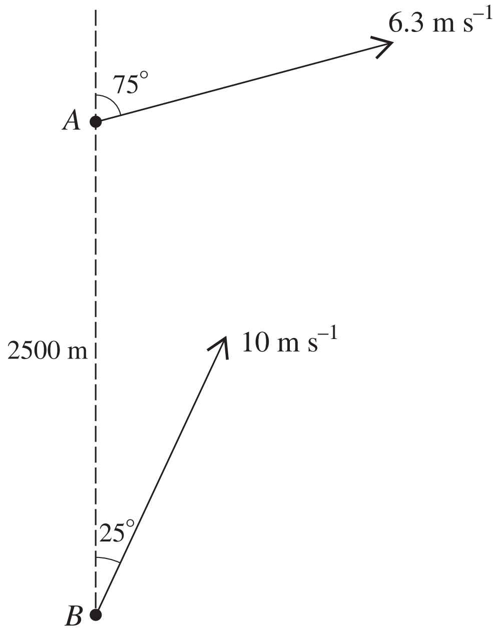
(i) Find the angular deceleration of the turntable. [2]

(ii) Find the angular speed of the turntable at the start of its last complete revolution before coming to rest. [2]

(iii) Find the time taken for the turntable to make its last complete revolution before coming to rest. [2]

3 The region bounded by the curve $y = 2x + x^2$ for $0 \leqslant x \leqslant 3$, the x-axis, and the line $x = 3$, is occupied by a uniform lamina. Find the coordinates of the centre of mass of this lamina. [9]

4



A boat A is travelling with constant speed $6.3\,\text{m s}^{-1}$ on a course with bearing 075°. Boat B is travelling with constant speed $10\,\text{m s}^{-1}$ on a course with bearing 025°. At one instant, A is 2500 m due north of B (see diagram).

(i) Find the magnitude and bearing of the velocity of A relative to B. [5]

(ii) Find the shortest distance between A and B in the subsequent motion. [3]

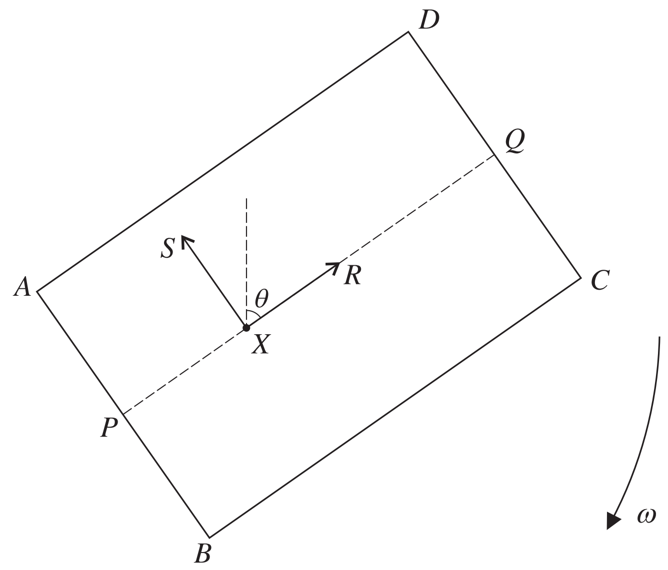
5 The region bounded by the curve $y = \sqrt{ax}$ for $a \leqslant x \leqslant 4a$ (where a is a positive constant), the x-axis, and the lines $x = a$ and $x = 4a$, is rotated through 2π radians about the x-axis to form a uniform solid of revolution of mass m.

(i) Show that the moment of inertia of this solid about the x-axis is $\frac{7}{5}ma^2$. [8]

The solid is free to rotate about a fixed horizontal axis along the line $y = a$, and makes small oscillations as a compound pendulum.

(ii) Find, in terms of a and g, the approximate period of these small oscillations. [4]

6



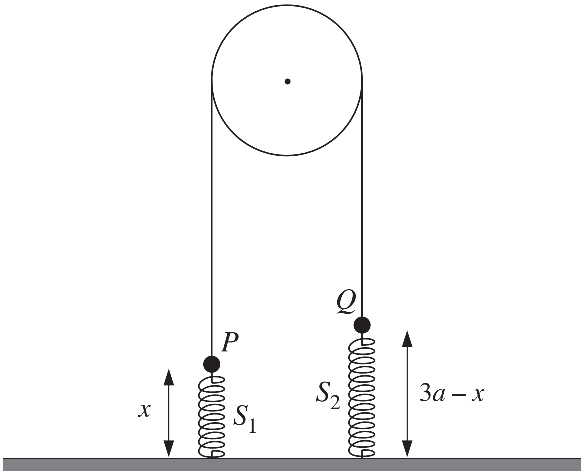
A uniform rectangular lamina $ABCD$ has mass m and sides $AB = 2a$ and $BC = 3a$. The mid-point of AB is P and the mid-point of CD is Q. The lamina is rotating freely in a vertical plane about a fixed horizontal axis which is perpendicular to the lamina and passes through the point X on PQ where $PX = a$. Air resistance may be neglected. When Q is vertically above X, the angular speed is $\sqrt{\frac{9g}{10a}}$. When XQ makes an angle θ with the upward vertical, the angular speed is ω, and the force acting on the lamina at X has components R parallel to PQ and S parallel to BA (see diagram).

(i) Show that the moment of inertia of the lamina about the axis through X is $\frac{4}{3}ma^2$. [3]

(ii) At an instant when $\cos\theta = \frac{3}{5}$, show that $\omega^2 = \frac{6g}{5a}$. [3]

(iii) At an instant when $\cos\theta = \frac{3}{5}$, show that $R = 0$, and given also that $\sin\theta = \frac{4}{5}$ find S in terms of m and g. [9]

7



Particles P and Q, with masses $3m$ and $2m$ respectively, are connected by a light inextensible string passing over a smooth light pulley. The particle P is connected to the floor by a light spring S_1 with natural length a and modulus of elasticity mg. The particle Q is connected to the floor by a light spring S_2 with natural length a and modulus of elasticity $2mg$. The sections of the string not in contact with the pulley, and the two springs, are vertical. Air resistance may be neglected. The particles P and Q move vertically and the string remains taut; when the length of S_1 is x, the length of S_2 is $(3a - x)$ (see diagram).

(i) Find the total potential energy of the system (taking the floor as the reference level for gravitational potential energy). Hence show that $x = \frac{4}{3}a$ is a position of stable equilibrium. [9]

(ii) By differentiating the energy equation, and substituting $x = \frac{4}{3}a + y$, show that the motion is simple harmonic, and find the period. [9]

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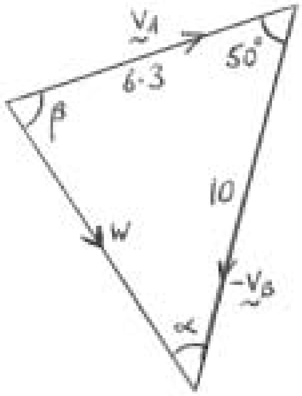
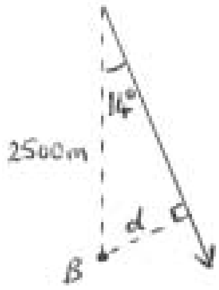
1	By conservation of angular momentum $1.5\times 21 + I_G\times 36 = 1.5\times 28 + I_G\times 34$ $I_G = 5.25 \text{ kg m}^2$	M1 A1A1 A1 **4**	Give A1 for each side of the equation or $1.5(28-21) = I_G(36-34)$
2 (i)	Using $\omega_1{}^2 = \omega_0{}^2 + 2\alpha\theta$, $0^2 = 8^2 + 2\alpha(2\pi\times 16)$ $\alpha = -\dfrac{1}{\pi} = -0.318$ Angular deceleration is 0.318 rad s^{-2}	M1 A1 **2**	*Accept* $-\dfrac{1}{\pi}$
(ii)	Using $\omega_1{}^2 = \omega_0{}^2 + 2\alpha\theta$, $\omega^2 = 8^2 + 2\alpha(2\pi\times 15)$ $\omega = 2 \text{ rad s}^{-1}$	M1 A1 ft **2**	or $0^2 = \omega^2 + 2\alpha(2\pi)$ ft is $\sqrt{64 - 60\pi\lvert\alpha\rvert}$ or $\sqrt{4\pi\lvert\alpha\rvert}$ *Allow A1 for* $\omega = 2$ *obtained using* $\theta = 16$ *and* $\theta = 15$ *(or* $\theta = 1$*)*
(iii)	Using $\omega_1 = \omega_0 + \alpha t$, $0 = \omega + \alpha t$ $t = 2\pi = 6.28 \text{ s}$	M1 A1 ft **2**	or $2\pi = 0t - \frac{1}{2}\alpha t^2$ ft is $\dfrac{\omega}{\lvert\alpha\rvert}$ or $\sqrt{\dfrac{4\pi}{\lvert\alpha\rvert}}$ *Accept* 2π
3	$A = \int_0^3 (2x + x^2)\,dx$ $= \left[x^2 + \frac{1}{3}x^3 \right]_0^3 = 18$ $A\bar{x} = \int_0^3 x(2x + x^2)\,dx$ $= \left[\frac{2}{3}x^3 + \frac{1}{4}x^4 \right]_0^3 = \dfrac{153}{4} = 38.25$ $\bar{x} = \dfrac{38.25}{18} = \dfrac{17}{8} = 2.125$ $A\bar{y} = \int_0^3 \frac{1}{2}(2x + x^2)^2\,dx$ $= \int_0^3 (2x^2 + 2x^3 + \frac{1}{2}x^4)\,dx$ $= \left[\frac{2}{3}x^3 + \frac{1}{2}x^4 + \frac{1}{10}x^5 \right]_0^3 = 82.8$ $\bar{y} = \dfrac{82.8}{18} = 4.6$	M1 A1 M1 M1 A1 M1 M1 M1 A1 **9**	*Definite integrals may be evaluated by calculator (i.e with no working shown)* Integrating and evaluating *(dependent on previous M1)* or $\int_0^{15} \left(3 - (\sqrt{y+1} - 1)\right) y\,dy$ Arranging in integrable form Integrating and evaluating *SR If ½ is missing, then M0M1M1A0 can be earned for* $\bar{y}$

<table>
<tr><td>4 (i)</td><td></td><td>B1</td><td>Correct velocity triangle</td></tr>
<tr><td></td><td>$w^2 = 6.3^2 + 10^2 - 2\times 6.3\times 10\cos 50°$
$w = 7.66 \text{ m s}^{-1}$
$\frac{\sin\alpha}{6.3} = \frac{\sin 50°}{w}$
$\alpha = 39.04°$ $(\beta = 90.96°)$
Bearing is $205 - \alpha = 166°$</td><td>M1
A1
M1

A1
5</td><td>

This mark cannot be earned from work done in part (ii)</td></tr>
<tr><td></td><td>OR $\begin{pmatrix}6.3\sin 75\\6.3\cos 75\end{pmatrix} - \begin{pmatrix}10\sin 25\\10\cos 25\end{pmatrix} = \begin{pmatrix}1.859\\-7.433\end{pmatrix}$ M1A1
M1
$w = \sqrt{1.859^2 + 7.433^2} = 7.66$ A1
Bearing is $180 - \tan^{-1}\frac{1.859}{7.433} = 166°$ A1</td><td></td><td>
Finding magnitude or direction</td></tr>
<tr><td>(ii)</td><td>As viewed from B

$d = 2500\sin 14.04$
$= 607$ m</td><td>B1 ft

M1
A1
3</td><td>Diagram showing path of A as viewed from B May be implied
Or B1 for a correct (ft) expression for d^2 in terms of t
or other complete method
Accept 604.8 to 609
SR If $\beta = 89°$ is used, give A1 for 684.9 to 689.1</td></tr>
</table>

5 (i)	$V = \int_a^{4a} \pi(ax)\,dx$	M1	*(Omission of π is an accuracy error)*
	$= \left[\frac{1}{2}\pi a x^2 \right]_a^{4a} = \frac{15}{2}\pi a^3$	M1	
	Hence $m = \frac{15}{2}\pi a^3 \rho$	M1	
	$I = \sum \frac{1}{2}(\rho\pi y^2 \delta x) y^2 = \int \frac{1}{2}\rho\pi y^4\,dx$	M1 A1	For $\int y^4\,dx$
	$= \int_a^{4a} \frac{1}{2}\rho\pi a^2 x^2\,dx$	A1 ft	Substitute for y^4 and correct limits
	$= \left[\frac{1}{6}\rho\pi a^2 x^3 \right]_a^{4a} = \frac{21}{2}\rho\pi a^5$	A1	
	$= \frac{7}{5}(\frac{15}{2}\pi a^3 \rho)a^2 = \frac{7}{5}ma^2$	A1 (ag) **8**	
(ii)	MI about axis, $I_A = \frac{7}{5}ma^2 + ma^2$	M1	Using parallel axes rule
	$= \frac{12}{5}ma^2$	A1	
	Period is $2\pi\sqrt{\frac{I}{mgh}}$	M1	
	$= 2\pi\sqrt{\frac{\frac{12}{5}ma^2}{mga}} = 2\pi\sqrt{\frac{12a}{5g}}$	A1 ft **4**	ft from any I with $h = a$
6 (i)	$I = \frac{1}{3}m\{a^2 + (\frac{3}{2}a)^2\} + m(\frac{1}{2}a)^2$	M1 M1	MI about perp axis through centre Using parallel axes rule
	$= \frac{13}{12}ma^2 + \frac{1}{4}ma^2 = \frac{4}{3}ma^2$	A1 (ag) **3**	
(ii)	By conservation of energy	M1	Equation involving KE and PE
	$\frac{1}{2}(\frac{4}{3}ma^2)\omega^2 - \frac{1}{2}(\frac{4}{3}ma^2)\frac{9g}{10a} = mg(\frac{1}{2}a - \frac{1}{2}a \times \frac{3}{5})$	A1	
	$\frac{2}{3}ma^2\omega^2 - \frac{3}{5}mga = \frac{1}{5}mga$		
	$\omega^2 = \frac{6g}{5a}$	A1 (ag) **3**	
(iii)	$mg\cos\theta - R = m(\frac{1}{2}a)\omega^2$	M1	Acceleration $r\omega^2$ and three terms (one term must be R)
	$mg \times \frac{3}{5} - R = \frac{3}{5}mg$	A1	*SR* $mg\cos\theta + R = m(\frac{1}{2}a)\omega^2 \Rightarrow R = 0$
	$R = 0$	A1 (ag)	earns M1A0A1
	$mg(\frac{1}{2}a\sin\theta) = I\alpha$	M1A1	Applying $L = I\alpha$
	$\alpha = \frac{3g}{10a}$	A1	
	$mg\sin\theta - S = m(\frac{1}{2}a)\alpha$	M1A1	Acceleration $r\alpha$ and three terms (one term must be S)
	$S = \frac{4}{5}mg - \frac{3}{20}mg$		or $S(\frac{1}{2}a) = I_G\,\alpha = \frac{13}{12}ma^2\alpha$
	$= \frac{13}{20}mg$	A1 **9**	

7 (i)	$U = 3mgx + 2mg(3a - x)$ $+\frac{mg}{2a}(x-a)^2 + \frac{2mg}{2a}(2a-x)^2$ $= \frac{mg}{2a}(3x^2 - 8ax + 21a^2)$ $\frac{dU}{dx} = 3mg - 2mg + \frac{mg}{a}(x-a) - \frac{2mg}{a}(2a-x)$ $= \frac{3mgx}{a} - 4mg$ When $x = \frac{4}{3}a$, $\frac{dU}{dx} = 4mg - 4mg = 0$ so this is a position of equilibrium $\frac{d^2U}{dx^2} = \frac{3mg}{a}$ > 0, so equilibrium is stable	B1B1 B1B1 M1 A1 A1 (ag) M1 A1 (ag) **9**	*Can be awarded for terms listed separately* Obtaining $\frac{dU}{dx}$ *(or any multiple of this)*
(ii)	KE is $\frac{1}{2}(3m)v^2 + \frac{1}{2}(2m)v^2$ Energy equation is $U + \frac{5}{2}mv^2 =$ constant Differentiating with respect to t $\left(\frac{3mgx}{a} - 4mg\right)\frac{dx}{dt} + 5mv\frac{dv}{dt} = 0$ $\frac{3gx}{a} - 4g + 5\frac{d^2x}{dt^2} = 0$ Putting $x = \frac{4}{3}a + y$, $\frac{3gy}{a} + 5\frac{d^2y}{dt^2} = 0$ $\frac{d^2y}{dt^2} = -\frac{3g}{5a}y$ Hence motion is SHM with period $2\pi\sqrt{\frac{5a}{3g}}$	M1A1 M1 A1 ft A1 ft M1A1 ft A1 (ag) A1 **9**	 Differentiating the energy equation (with respect to t or x) *Condone $\ddot{x}$ instead of $\ddot{y}$* *Award M1 even if KE is missing* *Must have $\ddot{y} = -\omega^2 y$ or other satisfactory explanation*